

# Variable-Speed Pump Efficiency Calculation For Fluid Flow Systems with and without Static Head

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## ABSTRACT

To accurately calculate pump energy savings gained from implementing variable frequency drive (VFD) controls, the variation of pump efficiency must be considered when operating conditions transition from the design operating point to new operating points. Many software tools require users to specify the new pump efficiency, or it is assumed to be unchanged. Unfortunately, many users have challenges of estimating the pump efficiency at new operating points.

This article presents a simplified method of estimating centrifugal pump efficiency at new operating speeds when the pump is controlled by a VFD. This methodology applies to systems with and without static head when the system curve is not affected by the change, and also systems where the change in operation changes the system curve.

A hypothetical fluid flow system and centrifugal pump were used to demonstrate the calculation process for these scenarios. For this hypothetical system, the pump's efficiency at new operating points was up to 5.4% lower than the design operating point.

## INTRODUCTION

Pump systems are ubiquitous in manufacturing facilities, water and wastewater plants, and commercial buildings. Pump systems transfer various types of fluids to provide heating, cooling, motive forces and materials needed for buildings and processes. In the manufacturing sector of the U.S., about 27% of electricity was used by pumps [1]. Many technical resources [1, 2] and training opportunities [3, 4] are available for facility managers to improve pump efficiency.

When operating conditions require multiple operating head and flow rate combinations, the most frequently recommended pump energy conservation measures in energy assessments are to install a variable frequency drive (VFD) and slow down the pump speed instead of riding the pump curve [5].

To calculate the pump energy savings from implementing VFD controls, in addition to the measured flow rate and head at the new operating point, the pump's efficiency at the new operating point is also required [6]. The pump's efficiency at the new operating point can be very different from the efficiency at the design operating point [1]. Unfortunately, some software tools simply assume that the pump efficiency does not vary unless the user specifies a different value [7], but many users have difficulties in estimating the new pump efficiency. This article describes how to estimate the variable speed pump efficiency for three possible systems: no static head and no changes to system curve; with static head and no changes to system curve; with static head and changes to system curve.

A hypothetical fluid flow system and centrifugal pump were used to demonstrate the calculation process for these three scenarios.

These calculations address changes in pump efficiency resulting from speed control only. Other issues such as net positive suction head available (NPSHA) and minimum continuous stable flow (MCSF) must be evaluated when implementing speed control of centrifugal pumps.

## VARIABLE SPEED PUMP EFFICIENCY CALCULATION

### Systems without Static Head

For a system with no static head (Figure 1), typically in closed loop systems, the pump operates at constant efficiency under variable speed control [8].

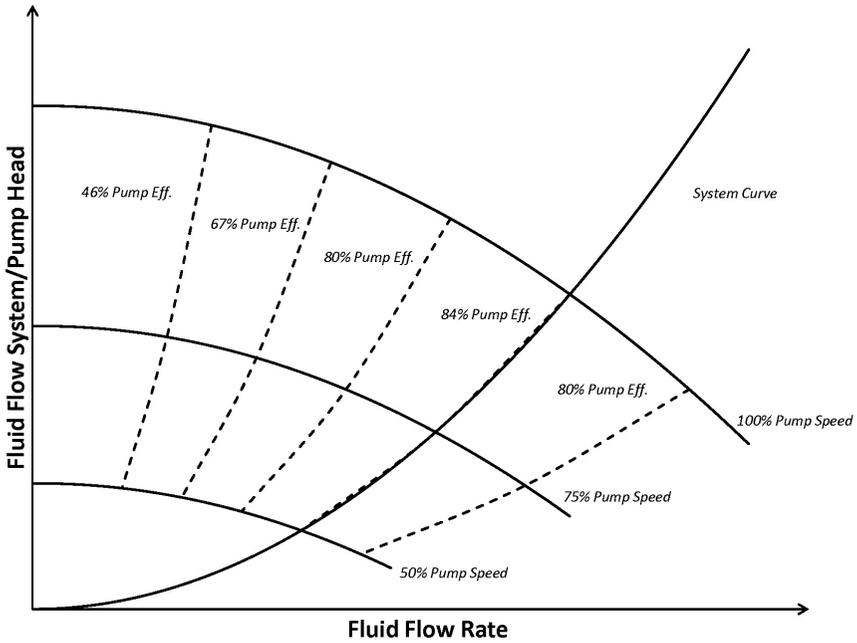
According to the affinity law, the new operating speed,  $S\%$ , can be obtained by using Equation 1.

$$S\% = \frac{GPM'}{GPM} \quad (1)$$

where GPM is the design flow rate and GPM' is the new operating flow rate.

For most centrifugal pumps, when the new operating speed is greater than 66.7% of full speed, it is typically acceptable to assume that the pump efficiency at the new operating point is the same as the efficiency at the design operating point [9], as shown in Equation 2.

$$\eta' = \eta \quad (2)$$



**Figure 1. Fluid Flow System without Static Head**

where  $\eta$  is the pump efficiency at the design, and  $\eta'$  is the efficiency at the new operating point.

When the new operating speed is less than 66.7% of full speed, the pump efficiency degradation caused by speed variation can be expressed as Equation 3 [9]. It should be noted that operation below the minimum continuous stable flow (MCSF) is not recommended.

$$\eta' = 1 - (1 - \eta) \left( \frac{1}{S\%} \right)^{0.1} \quad (3)$$

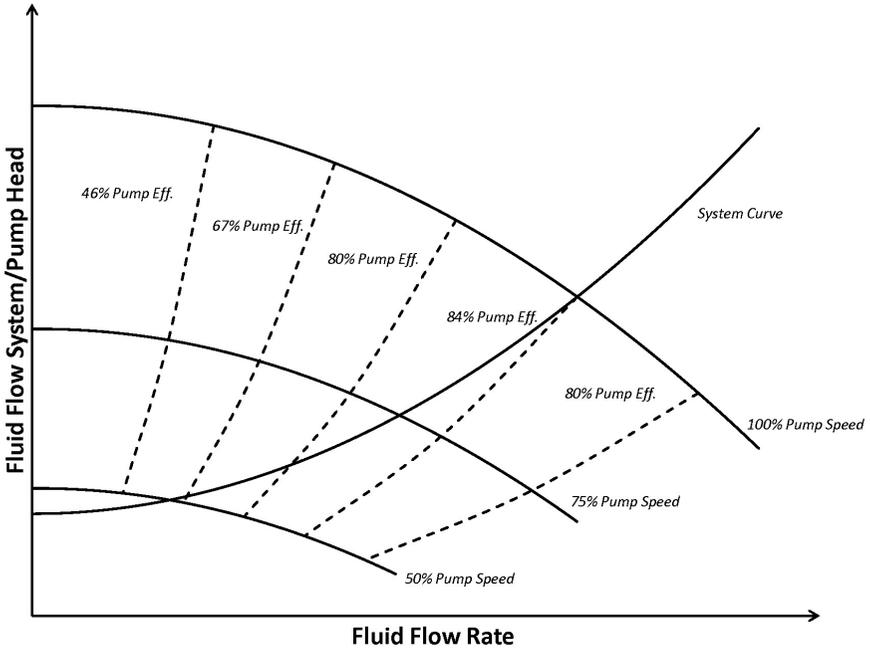
Combining Equations 1 and 3 results in Equation 4.

$$\eta' = 1 - (1 - \eta) \left( \frac{GPM}{GPM'} \right)^{0.1} \quad (4)$$

When the new operating speed is less than 66.7%, the pump efficiency can be obtained by using Equation 4.

### **Systems with Static Head**

For systems with static head (Figure 2), the pump does not maintain constant efficiency when operated with speed control [8].



**Figure 2. Fluid Flow System with Static Head**

For this case, the calculation procedure is described in the three steps presented below. Step 1 is to determine the required pump operating speed for the new operating point, Step 2 is to calculate the nominal flow rate with the same pump efficiency as at the new operating point, and Step 3 is to use the nominal flow rate and nominal pump efficiency curve to determine the pump efficiency at the new operating point.

This algorithm requires quadratic curve fits for pump head and efficiency. The curve fits can be directly provided by the user, or they can be derived from multiple performance data points.

*Step 1: Determine the required pump speed for the new operating point*

Assume that the pump head and flow relationship at the nominal or 100% speed can be presented in a quadratic equation, as in Equation 5 [10].

$$H = a + b \times GPM + c \times GPM^2 \tag{5}$$

At the new operating speed, S%, the head and flow rate are designated as H' and GPM'. According to the affinity law, the relationships between H' and GPM' and H and GPM are shown in Equations 6 and 7.

$$\frac{H'}{H} = S\%^2 \quad (6)$$

$$\frac{GPM'}{GPM} = S\% \quad (7)$$

Apply function transformation by plugging Equations 6 and 7 into Equation 5.

$$\frac{H'}{S\%^2} = a + b \times \left(\frac{GPM'}{S\%}\right) + c \times \left(\frac{GPM'}{S\%}\right)^2 \quad (8)$$

Rearrange Equation 8, and Equation 9 will be obtained:

$$a \times (S\%)^2 + b \times GPM' \times (S\%) + c \times GPM'^2 - H' = 0 \quad (9)$$

With the measured flow rate and pump head at the new operating point, Equation 10 can be obtained by solving Equation 9, and the result can be used to obtain S% [11].

$$S\% = \frac{-b \times GPM' + \sqrt{(b \times GPM')^2 - 4 \times a \times (c \times GPM'^2 - H')}}{2 \times a} \quad (10)$$

*Step 2: Determine the flow rate at the nominal or 100% speed with the same pump efficiency as at the new operating point*

Based on the affinity law, the iso-efficiency lines for variable speeds follow Equation 11. In other words, the  $\eta$  and  $\eta'$  for GPM and GPM' are the same.

$$GPM = \frac{GPM'}{S\%} \quad (11)$$

*Step 3: Determine the pump efficiency at the new operating point*

Assume the pump efficiency curve at the nominal or 100% speed can be presented in a quadratic equation, as in Equation 12.

$$\eta = f \times GPM + g \times GPM^2 \quad (12)$$

Equation 13 can be obtained by combining Equations 11 and 12.

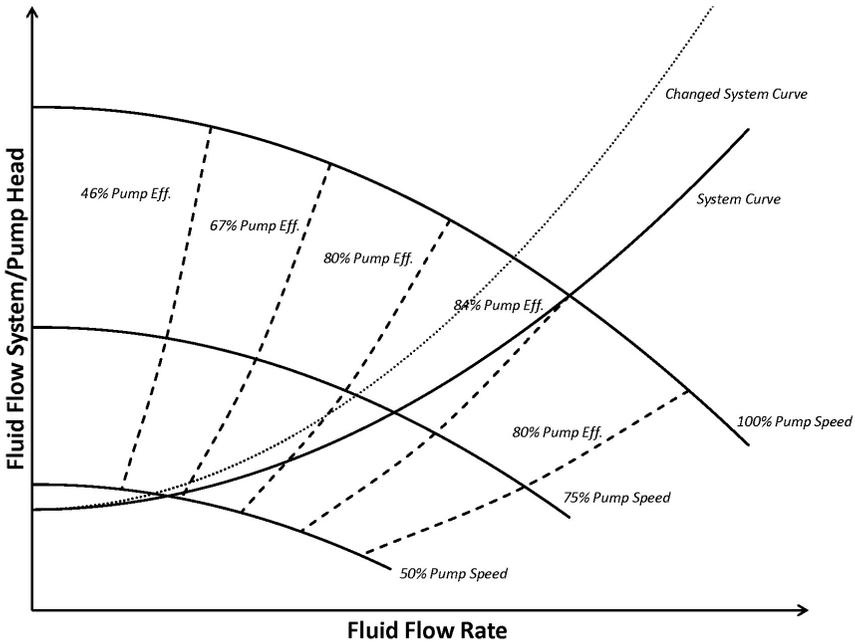
$$\eta' = f \times \frac{GPM'}{S\%} + g \times \left(\frac{GPM'}{S\%}\right)^2 \quad (13)$$

When the new operating speed is greater than 66.7%, the pump efficiency can be obtained by using Equation 13. When the new operating speed is less than 66.7%, the pump efficiency can be obtained by using Equation 14, with the consideration of the pump efficiency degradation caused by speed variation, as in Equation 3 above [9].

$$\eta' = 1 - \left( 1 - f \times \frac{GPM'}{S\%} - g \times \left( \frac{GPM'}{S\%} \right)^2 \right) \left( \frac{1}{S\%} \right)^{0.1} \tag{14}$$

**Systems with Static Head and Changed System Curve**

Changes to the resistance to flow in a system will change the relationship between flow rate and head and will manifest as changes to the system curve. This change can result from changes in valve position, flow path, equipment on-line (e.g., number of chillers, heat exchanges, or cooling towers being served) (Figure 3).



**Figure 3. Fluid Flow System with Static Head and Changed System Curve**

The three-step calculation described above is agnostic to the change of the system curve (i.e., system flow rate and head relationship). Therefore, Equations 13 and 14 can be also used for this scenario.

## SAMPLE CALCULATIONS

Table 1 shows the data points for the flow rate, pump head, and pump efficiency at 100% speed for a hypothetical centrifugal pump. The pump head curve fit was generated as Equation 15, and the pump efficiency curve fit was generated as Equation 16:

$$H = 50 + 0.0004 \times GPM - 0.000006 \times GPM^2 \quad (15)$$

$$\eta = 0.0009288 \times GPM - 0.000000258 \times GPM^2 \quad (16)$$

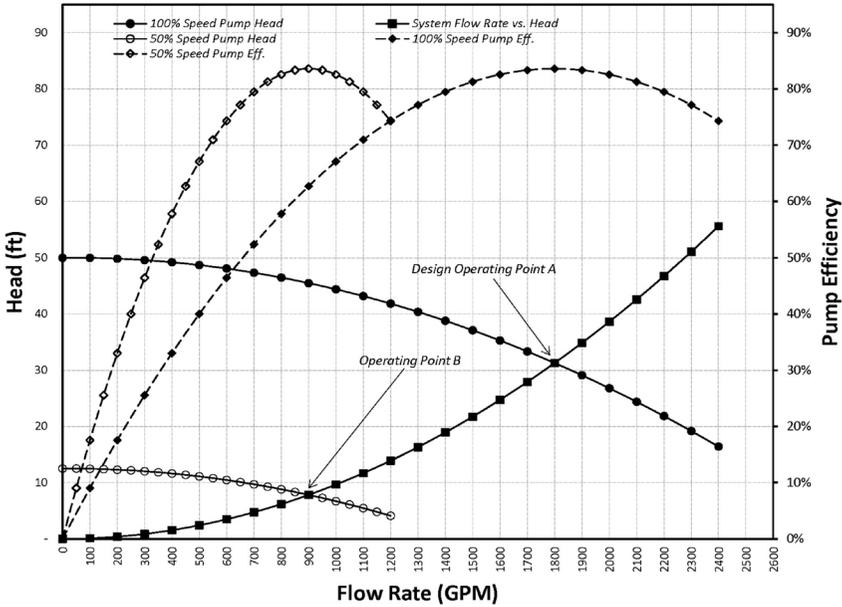
At the design operating point A, the flow rate is 1,800 GPM, the head is 31.3 ft w.g., and the pump efficiency is 83.6%. Three cases are presented below: one system without static head, one with static head, and one with static head and changed system curve.

**Table 1. Pump Flow Rate, Head, and Efficiency Data Points at 100% Pump Speed**

Pump Flow Rate (GPM)	Pump Head (ft w.g.)	Pump Efficiency
0	50.0	0.0%
100	50.0	9.0%
200	49.8	17.5%
300	49.6	25.5%
400	49.2	33.0%
500	48.7	40.0%
600	48.1	46.4%
700	47.3	52.4%
800	46.5	57.8%
900	45.5	62.7%
1,000	44.4	67.1%
1,100	43.2	71.0%
1,200	41.8	74.3%
1,300	40.4	77.1%
1,400	38.8	79.5%
1,500	37.1	81.3%
1,600	35.3	82.6%
1,700	33.3	83.3%
<b>1,800</b>	<b>31.3</b>	<b>83.6%</b>
1,900	29.1	83.3%
2,000	26.8	82.6%
2,100	24.4	81.3%
2,200	21.8	79.5%
2,300	19.2	77.1%
2,400	16.4	74.3%

**Case Study 1: System without Static Head**

The flow rate of the new operating point B is 900 GPM, and the head is 7.8 ft w.g. The system curve, the design operating point A, and the new operating point B are presented in Figure 4. The pump head and efficiency curves at new operating speeds were created using the affinity law and are also included in Figure 4 to validate the mathematically calculated new pump efficiency.



**Figure 4. System without Static Head**

Because this system has no static head, Equation 1 was used to calculate the new operating pump speed:

$$S\% = \frac{GPM'}{GPM} = \frac{900}{1800} = 50\%$$

Because the new operating pump speed is less than 66.7%, Equation 4 was used to calculate the pump efficiency at operating point B.

$$\eta' = 1 - (1 - 83.6\%) \left(\frac{1800}{900}\right)^{0.1} = 82.4\%$$

The pump efficiency at operating point B is 1.2% lower than at design operating point A.

### Case Study 2: System with Static Head

The flow rate of the new operating point C is 900 GPM and the head is 15.3 ft w.g. Figure 5 presents the system flow rate vs. the head curve, the design operating point A, and the new operating point C. Figure 5 also includes the pump head and efficiency curves at the new operating speed, which were created using the affinity law to validate the mathematically calculated new pump efficiency.

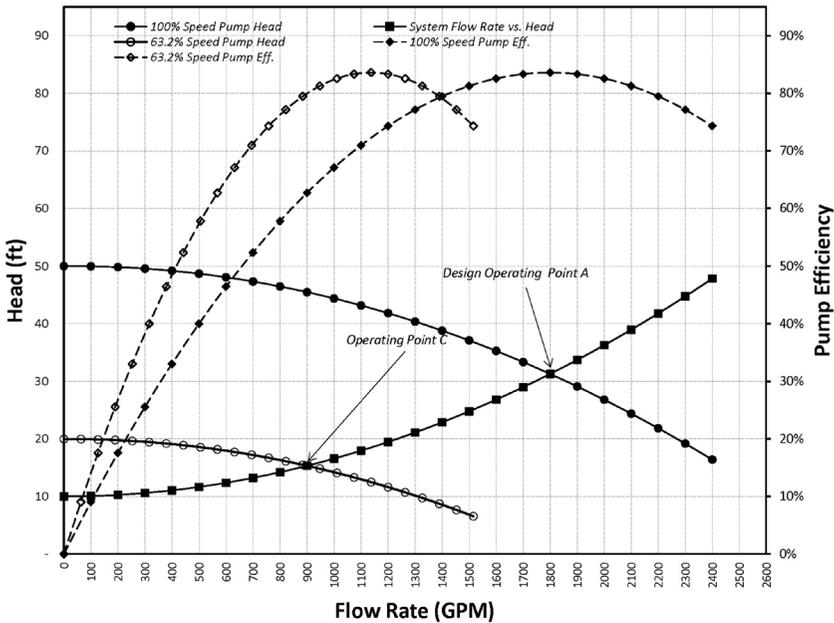


Figure 5. System with Static Head

Using Equation 10 to calculate the new operating pump speed,

$$S\% = \frac{-0.0004 \times 900 + \sqrt{(0.0004 \times 900)^2 - 4 \times 50 \times (-0.000006 \times 900^2 - 15.3)}}{2 \times 50} = 63.2\%$$

Because the new pump speed is lower than 66.7%, Equation 14 was used to calculate the pump efficiency at the new operating point.

$$\eta' = 1 - \left( 1 - \left( 0.0009288 \times \frac{900}{63.2\%} - 0.000000258 \times \left( \frac{900}{63.2\%} \right)^2 \right) \right) \left( \frac{1}{63.2\%} \right)^{0.1} = 79.0\%$$

The pump efficiency at operating point C is 4.6% lower than at the design operating point A.

### Case Study 3: System with Static Head and Changed System Curve

The flow rate of the new operating point D is 900 GPM, and the head is 17.8 ft w.g. The system curves before and after the changed flow rate, the design operating point A, and the new operating point D are presented in Figure 6. To validate the mathematically calculated new pump efficiency, the pump head and efficiency curves at the new operating speed were created using the affinity law and are included in Figure 6.

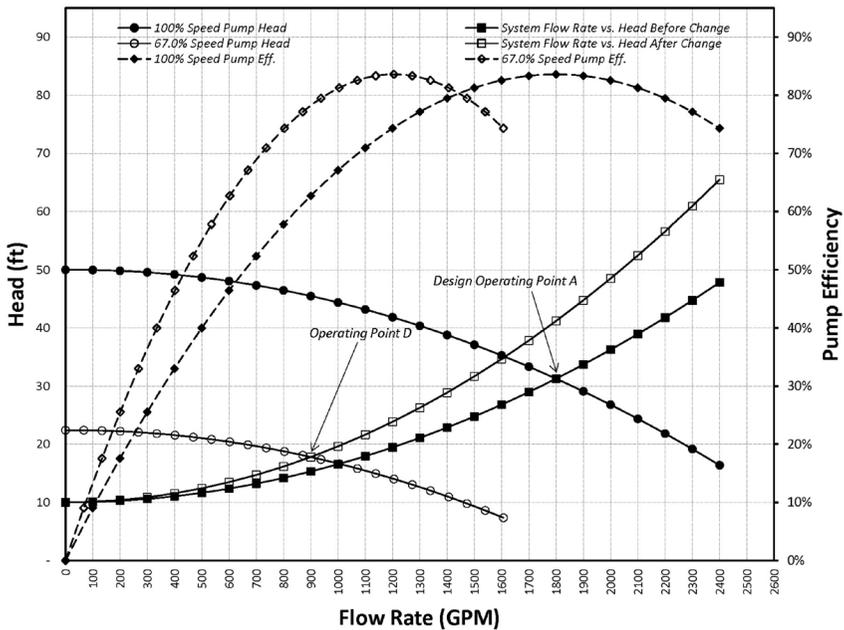


Figure 6. System with Static Head and Changed System Curve

Using Equation 10 to calculate the new operating pump speed,

$$S\% = \frac{-0.0004 \times 900 + \sqrt{(0.0004 \times 900)^2 - 4 \times 50 \times (-0.000006 \times 900^2 - 17.8)}}{2 \times 50} = 67.0\%$$

Because the new pump speed is higher than 66.7%, Equation 13 was used to calculate the pump efficiency at the new operating point:

$$\eta' = 0.0009288 \times \frac{900}{67.0\%} - 0.000000258 \times \left(\frac{900}{67.0\%}\right)^2 = 78.2\%$$

The pump efficiency at operating point D is 5.4% lower than at the design operating point A.

## CONCLUSIONS

When using VFD controls to reduce pump energy consumption, the pump efficiency at the new operating point is required to accurately calculate the pump energy savings. This article provides a procedure on how to estimate the new pump efficiency for three possible scenarios: systems without static head, systems with static head, and systems with static head and changed system curve. The calculation procedure is very easy for users to implement in Excel spreadsheet calculators and in modern, stand-alone software, or it can be used to enhance currently existing software tools to obtain more accurate pump energy savings results. For the hypothetical fluid flow system in the case studies, the pump efficiency at new operating points was up to 5.4% lower than at the design operating point.

## ACKNOWLEDGMENTS

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